ISSN 1069-3513, Izvestiya, Physics of the Solid Earth, 2009, Vol. 45, No. 8, pp. 701–708. © Pleiades Publishing, Ltd., 2009.
Original Russian Text © M.Yu. Reshetnyak, 2009, published in Fizika Zemli, 2009, No. 8, pp. 83–90.

On the Locality of the Kinetic Energy Transfer in a Wave Space

M. Yu. Reshetnyak

Schmidt Institute of Physics of the Earth, Russian Academy of Sciences, Bol'shaya Gruzinskaya ul. 10, Moscow, 123995 Russia

Received July 16, 2008

Abstract—For the model of thermal turbulence in the Boussinesq approximation with heating from below a change in the structure of the nonlinear interaction of the harmonics of the velocity field with the appearance of rotation is examined. The parameters of convection with rotation are chosen in such a way to correspond to the typical regimes in the models of the planetary dynamo. For such regimes, the structure of the triadic mechanism of the kinetic energy transfer through the spectrum is investigated.

Key words: geodynamo, cascade processes, turbulence, triads.

PACS numbers: 91.45.Ku

DOI: 10.1134/S1069351309080096

INTRODUCTION

According to the classical Kolmogorov model for sufficiently high amplitude of energy sources in liquids, gases, and plasma, the energy cascade (the Richardson cascade) appears. Depending on the system dimensionality kinetic energy is transferred both from the large scales to the smaller scales (the direct cascade of energy for 3D turbulence) and in the opposite direction (the inverse cascade of energy for 2D turbulence). This model obtained confirmation both in numerous numerical experiments [Tabeling, 2002] and in the theory of renormalization groups [Kraichnan and Montgomery, 1980].

In the regime of pure convection, for example, in the problem with the induced force, the energy transfer occurs due to the nonlinear term in the Navier–Stokes equations. Since the nonlinear term is quadratic in the field of velocity **V**, it can be called the triadic interaction [Pedlosky, 1987]. In the general case, the wave with the wave vector $k = \pm|\mathbf{k}|$ is formed due to the interaction of the pair of waves **p** and **q** so that $\mathbf{k} + \mathbf{p} + \mathbf{q} = 0$, where at least two wave numbers are close in value. The local ($k \sim p \sim q$) and nonlocal ($k \ll p$, $p \sim q$) interactions are distinguished. If energy is transferred from q to k and $k \sim q$ ($k \ll q$, $k \gg q$), then it can be called the local (nonlocal) energy transfer. The locality of interaction results in the local energy transfer while, generally speaking, the converse is false [Alexakis et al., 2007]. Despite the fact that harmonics with greatly differing wave vectors can participate in nonlinear interaction, for nonrotating turbulence the contribution of harmonics with the close wave numbers predominates (~80%, for the Reynolds numbers Re ~800), and energy is transferred locally through the spectrum (the contribution of other harmonics is weakly correlated). The remaining 20% that correspond to the nonlocal interaction are sufficient for the appearance of the long characteristic times (considerably longer than the turn-round time of vortex) in the small scales.

In passing from the problem of convection with the induced force to the task of heat convection with the Prandtl numbers Pr ~1, the picture of energy transfer, on the whole, does not change, although a certain decrease of the nonlocal energy transfer from the scale of the induced force to the small scales is possible. The picture radically changes, when the system is set into rotation. In this case, the Rossby waves appear in the liquid medium [Pedlosky, 1987], that have the small scale in the plane, which is perpendicular to the axis of rotation, and have the large scale along the axis of rotation. In the zero approximation the so-called geostrophic balance appears, which is reduced to the balance of forces of pressure and the Coriolis force. In the next order in the expansion in the Ekman number the Coriolis force demonstrates an anticorrelation with the nonlinear term and blocks the energy transfer through the spectrum. Formally, this mechanism can be understood as follows: the nonlinear term in the Navier–Stokes equations can be written in the form $\mathbf{V} \times (\nabla \times \mathbf{V})$. However, the introduction of rotation leads to the appearance of the hydrodynamic helicity: $\chi = \mathbf{V} \cdot (\nabla \times \mathbf{V})$ (the mean with respect to volume $\langle\chi\rangle$ without rotation is equal to zero). It is obvious that by changing the angle between **V** and the vorticity of the field of velocity $\omega = \nabla \times \mathbf{V}$, it is possible to attain an increase in one value, with a decrease in another one. The analogy with the magnetic field is of interest: it is known that the large-scale magnetic field also attenuates the cascade of kinetic energy, allowing the energy flow form the large scales to the small scales directly (the nonlocal cascade) [Alexakis et al., 2007]. In both cases the formation of 2D flow occurs due to the magnetic field, or the forces of rotation [Batchelor, 1953].

The noted formation of 2D flows leads to the appearance of inverse cascades of energy in the problems of heat convection and the dynamo with rotation [Reshetnyak, 2008a]. Since in the large scales the extraction of energy is less efficient than in the small scales (where dissipation occurs), in the real quasi-geostrophic systems the amplitude of the inverse flow of energy is not great. The direct 3D calculations give evidence of the appearance of regimes, which are close to the statistical equilibrium. The latter implies not only the statistical stationarity of random fields, but also the absence of the fluxes of these values in the wave space (i.e., there is no energy exchange between the scales) [Rose and Sulem, 1978]. For example it can be noted that the Kolmogorov theory is based on the constancy of the kinetic energy flux within the inertia interval.

Further I will return to the results of the work [Reshetnyak, 2008b] and will consider the questions of triadic interaction and locality of the energy transfer in the wave space for the convection in the rotating rectangular box with the incompressible fluid heated from below.

THE CONVECTION EQUATIONS

Consider the heat convection of incompressible fluid $\nabla \cdot \mathbf{V} = 0$ in the rectangular box rotating with an angular velocity Ω relative to the vertical axis $\mathbf{z}$. After introducing the following units of measurement for velocity $\mathbf{V}$, time t, and pressure P: κ/L, L^2/κ and $\rho\kappa^2/\mathrm{L}^2$, where L is the unit of length, κ is the coefficient of molecular thermal conductivity, and ρ is the substance density, let us write the system of equations of the dynamo in the Cartesian system of coordinates (x, y, z) in the form:

$$\mathrm{E}\,\mathrm{Pr}^{-1}\left[\frac{\partial \mathbf{V}}{\partial t} - \mathbf{V}\times(\nabla\times\mathbf{V})\right] = (-\nabla P - \mathbf{1_z}\times\mathbf{V} + \mathrm{Ra}T\mathbf{1_z} + \mathrm{E}\Delta\mathbf{V}), \qquad (1)$$
$$\frac{\partial T}{\partial t} + (\mathbf{V}\cdot\nabla)(T + T_0) = \Delta T.$$

The dimensionless Prandtl, Ekman, Rayleigh, and Roberts numbers are introduced as: $\mathrm{Pr} = \frac{\nu}{\kappa}$, $\mathrm{E} = \frac{\nu}{2\Omega \mathrm{L}^2}$, and $\mathrm{Ra} = \frac{\alpha g_0 \delta T L}{2\Omega\kappa}$, where ν is the kinematic viscosity coefficient, α is the coefficient of volumetric expansion, g_0 is the gravity acceleration, δT is the unit of disturbance of temperature T relative to the "diffusion" temperature distribution of $T_0 = 1 - z$. The Rossby number is introduced as $\mathrm{Ro} = \mathrm{EPr}^{-1}$. Problem (1) was solved in the rectangular box with the periodic boundary conditions for coordinates x and y. For the boundaries $z = 0, 1$ the disturbances of temperature T were equal to zero, which taking into account the selected profile of T_0, is equivalent to the assignment of temperatures on the boundaries: $\tilde{T} = T + T_0 = 1, 0$ (heating from below). For the velocity field, the condition of nonpenetration and equality to zero of the gradients of tangential components at $z = 0, 1$: $V_z = \frac{\partial V_x}{\partial z} = \frac{\partial V_y}{\partial z} = 0$ are assumed to be valid. The formulation of boundary conditions in such a way guarantees the equality to zero of the tangential components of the tensor of viscous stresses, and also the zero values of hydrodynamic helicity. For the solution of (1) the pseudo-spectral code was used (see details in [Orszag, 1971; Meneguzzi and Pouquet, 1989; Cattaneo et al., 2003]), adapted for parallel processors with MPI utilization [Reshetnyak, 2007]. The calculations were conducted on the grid of N^3, $N = 64$.

FLUXES IN k-SPACE

For describing the exchange interactions in the Fourier-space it is convenient to divide the wave space into shells, such, that $k_i < k < k_{i+1}$, $k_{i+1}/k_i = \gamma$, while it is usually accepted that $\gamma = 2$. Further, consider in the energy exchange between such shells (the cascade processes). Let us introduce the expansion of field f in the high and low-frequency components: $f(\mathbf{r}) = f^<(\mathbf{r}) + f^>(\mathbf{r})$, where

$$f^<(\mathbf{r}) = \sum_{|k|\le K} \hat{f}_k e^{i\mathbf{kr}}, \quad f^>(\mathbf{r}) = \sum_{|k|>K} \hat{f}_k e^{i\mathbf{kr}}, \qquad (2)$$

accordingly. For the periodic fields f and g the following conditions are true (see details in [Frisch, 1995]):

$$\left\langle\frac{\partial f}{\partial x}\right\rangle = 0, \quad \left\langle g\frac{\partial f}{\partial x}\right\rangle = -\left\langle f\frac{\partial g}{\partial x}\right\rangle, \quad \langle f^> g^<\rangle = 0, \qquad (3)$$

where

$$\langle f(\mathbf{r})\rangle = \mathrm{v}^{-1}\int_{\mathrm{v}} f(\mathbf{r})d\mathbf{r}^3 \qquad (4)$$

means the averaging of field f over volume V. Multiplying the Navier–Stokes equations by $\mathbf{V}^<$, one can obtain an expression for the average over the volume of a change of the kinetic energy in the sphere of radius K:

$$2^{-1}\mathrm{E}\,\mathrm{Pr}^{-1}\left[\frac{\partial\langle V_i^< V_i^<\rangle}{\partial t} + \Pi(K)\right] = \mathrm{Ra}\langle T^< V_z^<\rangle - \mathrm{E}\langle(\nabla\omega^<)^2\rangle, \qquad (5)$$

where the integral flux of kinetic energy from region $k > K$ into region $k \le K$ is assigned in the form

$$\Pi(K) = \langle V_i^< \cdot (V_j\cdot\nabla_j)V_i\rangle, \qquad (6)$$

and summation is conducted over the repetitive indices $i = 1\ldots, 3$. It is convenient to introduce the local flux of kinetic energy T_K:

$$T_K(k) = -\frac{\partial\Pi(K)}{\partial k}, \quad \int_{k=0}^{\infty} T_K(k)dk = 0. \qquad (7)$$

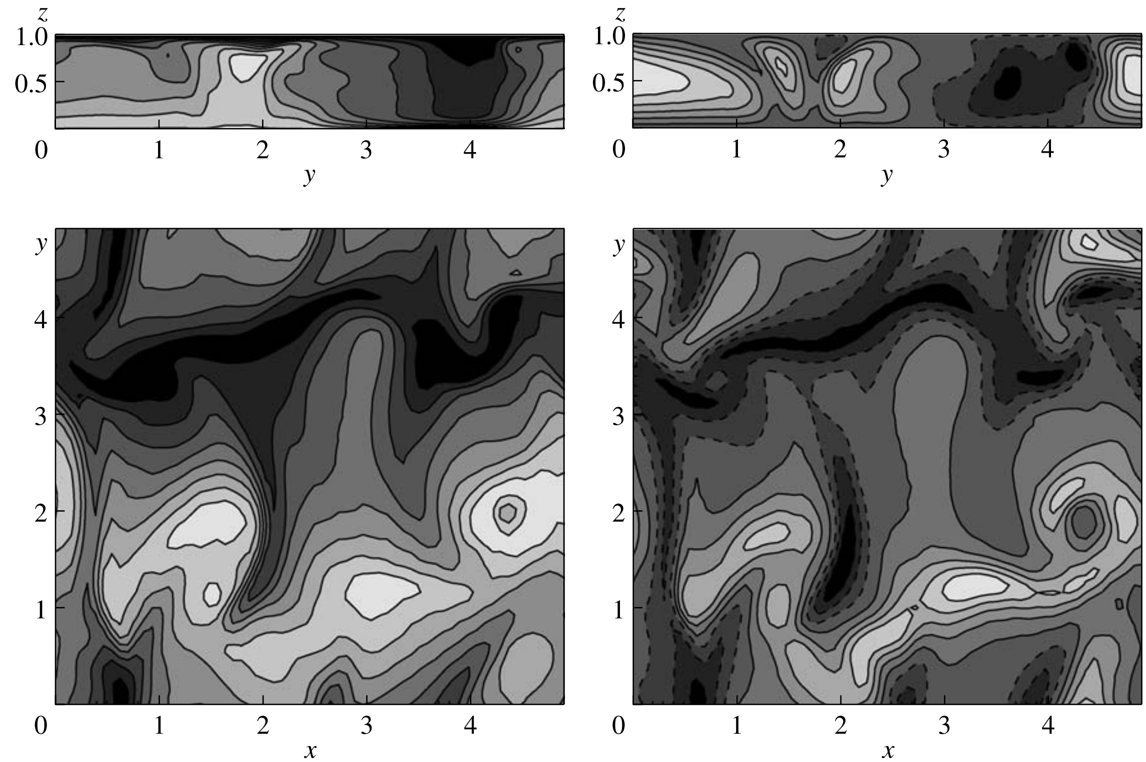

Fig 1. Sections of the temperature field T and the vertical component of velocity V_z without rotation (regime NR). The upper row is for $x = 4.3$, the lower row is for $z = 0.8$. The field ranges are (0, 1), (–257, 506) and (0.03, 0.86), (–254, 572).

Then, for the Navier–Stokes equation we have:

$$\frac{\partial E(K)}{\partial t} = T_K(k) + F(k) + D(k), \qquad (8)$$

where $E(k) = \frac{1}{2}\frac{\partial}{\partial k}\langle \mathbf{V}_k^2 \rangle$ is a change in the kinetic energy for the wave number k, $F(k) = \frac{\mathrm{Ra\,Pr}}{\mathrm{E}}\frac{\partial}{\partial k}\langle TV_z^< \rangle$ is the work of the force of Archimedes, and $D(k) = -\mathrm{Pr}k^2E(k)$ is the dissipation. Expression (8) describes the total flux of the kinetic energy through the wave number k.

For studying the detailed structure of the triadic mechanism it is possible to ask a question about the form of the energy equation, which describes the energy transfer from shell Q to shell K. Generally speaking, this formulation of the problem is nontrivial, since we always deal with three waves [Verma, 2004]. It is possible to show that such problem formulation is valid (see references in [Alexakis et al., 2007]):

$$\frac{\partial E(K)}{\partial t} = T_{uu} + A(K) + D(K), \qquad (9)$$

where $E(K)$ and $D(K)$ take the same form as in (8), and $T_{uu} = \langle V_i(K) \cdot (V_j \cdot \nabla_j)V_i(Q)\rangle$, $A(K) = \frac{\mathrm{Ra\,Pr}}{\mathrm{E}}\langle T(K)V_z(K)\rangle$. Here the flux of kinetic energy T_{uu} from shell Q to shell K depends on two wave numbers K and Q, respectively, that requires taking into account the fast Fourier transform utilized for multiplication of the nonlinear terms of the $N^5\ln(N)$ operations. The analysis of T_{uu} enables one to estimate whether the energy transfer is local or not, but not the locality of the interaction itself (the latter would require $N^6\ln(N)$ operations). In the general case, for the arbitrary periodic (or random uniform) nondivergent fields $\mathbf{u}(Q)$, $\mathbf{w}(K)$ and $\mathbf{V}$ we have [Alexakis et al., 2005]: $T_{uw}(Q, K) = -T_{wu}(K, Q)$, where $T_{uw}(Q, K) = \langle u_i(K) \cdot (V_j \cdot \nabla_j)w_i(Q)\rangle$, and $T_{wu}(K, Q) = \langle w_i(Q) \cdot (V_j \cdot \nabla_j)u_i(K)\rangle$, which corresponds to the equality of the energy obtained by shell K from shell Q to the energy returned by shell Q to shell K. In the next paragraph I will examine the properties of the fluxes T_K and T_{uu} by using the example of system (1) and their changes during the introduction of rotation.

THE CASCADE PROCESSES

Let us further briefly consider the results of the work [Reshetnyak, 2008b], that is necessary for understanding the subsequent material, and will proceed to a question about the locality of triadic interaction.

Consider three regimes of convection (Fig. 1):

NR: Regime without rotation (the Coriolis force vanishes), $\mathrm{Ra} = 9 \times 10^5$, $\mathrm{Pr} = 1$, $\mathrm{E} = 1$, and $\mathrm{Re} \sim 700$.

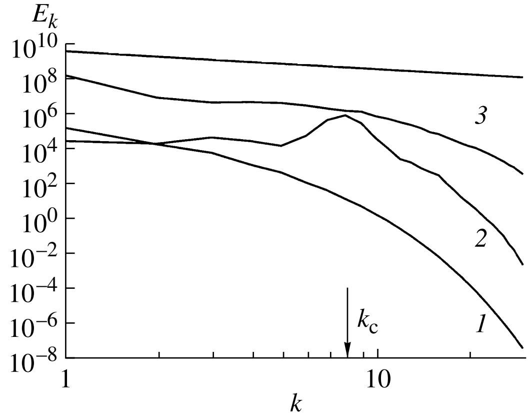

Fig. 2. Kinetic energy spectra for regimes (*1*) NR, (*2*) R1, and (*3*) R2. The straight line corresponds to the Kolmogorov spectrum with $\sim k^{-5/3}$.

R1: Regime with rotation, Ra = 4×10^2, Pr = 1, E = 2×10^{-5}, and Re ~ 200.

R2: Regime with rotation, Ra = 1×10^3, Pr = 1, E = 2×10^{-5}, and Re ~ 10^4.

The first regime, presented in Fig. 1, corresponds to turbulent convection without rotation.[1] The spectral properties of convection are close to the Kolmogorov dependence $\sim k^{-5/3}$ (Fig. 2), and the amplitude of the chaotic fluctuations of kinetic energy amounts to ~15% of its mean level.

Heat convection with rotation is characterized by the appearance of a large number of vertical rotating columns (cyclones–anticyclones). Their number depends on the Ekman number as $k_c \sim \mathrm{E}^{-5/3}$ [Chandrasekhar, 1961; Busse, 1970; Jones and Roberts, 2000]. For the liquid Earth core E ~ 10^{-15}, that obviously makes it impossible to conduct calculations with the realistic values of the parameters. Usually, it is possible to attain regimes with E = 10^{-4}–10^{-6} [Jones, 2000]. The purpose of numerical experiments is to obtain the asymptotic regime and to conduct the corresponding extrapolation to the Earth's parameters. Also, from linear analysis it is known that the critical Rayleigh number, with which convection begins, depends on the Ekman number as $\mathrm{Ra}^{cr} \sim \mathrm{E}^{-1/3}$. An increase in Ra^{cr} is connected with the appearance of cyclonic convection, which leads to increased dissipation.

Regime R1 corresponds to the geostrophic state of convection near the generation threshold, characterized by the regular spatial structure of cyclones (Fig. 3). An increase in Ra (regime R2) results in the disturbance of the ordering of cyclones, to the appearance of small-scale flows in the direction z, and to the deviation from geostrophism. The nonlinear term becomes closer by the amplitude to the Coriolis force and to the pressure gradient and the time behavior becomes more chaotic. In both cases with rotation the appearance of hydrodynamic helicity $\chi(z) = \langle \mathbf{V} \cdot \mathrm{rot}\mathbf{V} \rangle_{xy} \sim z - 0.5$ is observed (the averaging is carried out in the (x, y) plane) [Reshetnyak, 2007]. For the NR regime χ vanishes.

The spectra of convection with rotation differ from the regime without rotation. Regime R1 is close to the threshold. On the integral spectrum (the upper figure) the peak of $k_c \sim 8$ is clearly seen, which corresponds to cyclonic flows. It should be noted that for regime R1 the spectra of kinetic energy $E_K(k_\perp) = \int E_K(k_z)dk_z$ и $E_K(k_\parallel) = \iint E_K(k_x, k_y)dk_x dk_y$ considerably differ ($k_\perp^2 = k_x^2 + k_y^2$, $k_\parallel \equiv k_z$). For $k < k_c$ $E_K(k_\perp)$ is close to the white spectrum, and $E_K(k_\parallel) \sim k^{-3}$.

With an increase in Ra (regime R2), the spectrum notch begins to be filled and the integral spectrum becomes similar to the spectrum without rotation. The filling of the spectrum for the vertical part of the spectrum occurs more smoothly and it rapidly approaches the Kolmogorov dependence. The filling of the spectrum is also observed in problems with spherical geometry (see [Reshetnyak, 2006]) and in the cascade models of turbulence [Reshetnyak and Steffen, 2006].

The observed similarity in the behavior of spectra R2 and NR is not yet indicative of the similarity of the physical processes: it is known that in 2D turbulence [Kraiehnan and Montgomery, 1980] the spectrum of kinetic energy with $\sim k^{-5/3}$ is also observed, but the energy transfer proceeds not from the large scales to the smaller ones, but vice versa. The estimates of the terms for the R2 regime presented in [Reshetnyak, 2008] imply the fulfillment of the geostrophic balance.

Let us examine the behavior of $T_K(k)$. Regime NR demonstrates the well known picture of the Kolmogorov direct cascade of kinetic energy (Fig. 4). For the large scales $T_M^< < 0$ these scales are the energy sources. In passing to the infrared spectrum, the sign of flux changes and becomes positive: energy is consumed. For 2D turbulence, the reflection symmetric picture for the flux is observed [Kraiehnan and Montgomery, 1980]. In this case, instead of the direct cascade of energy the reverse cascade is observed.

Rotation significantly changes the behavior of energy fluxes. The energy-carrying wave number is k_c. For $k > k_c$ there is a direct cascade of energy $T_K^< > 0$. The larger the Re, the greater the maximum of $T_K^<$ is displaced to the right relative to the spectrum maximum. The picture is significantly more complex for $k < k_c$: for the small wave

[1] Since on the generating threshold the horizontal size of the convective cell is larger than the vertical size, the box elongated in the horizontal direction is usually used. In the calculations the following sizes were utilized: $L_x = L_y = 5L_z \equiv 5L$.

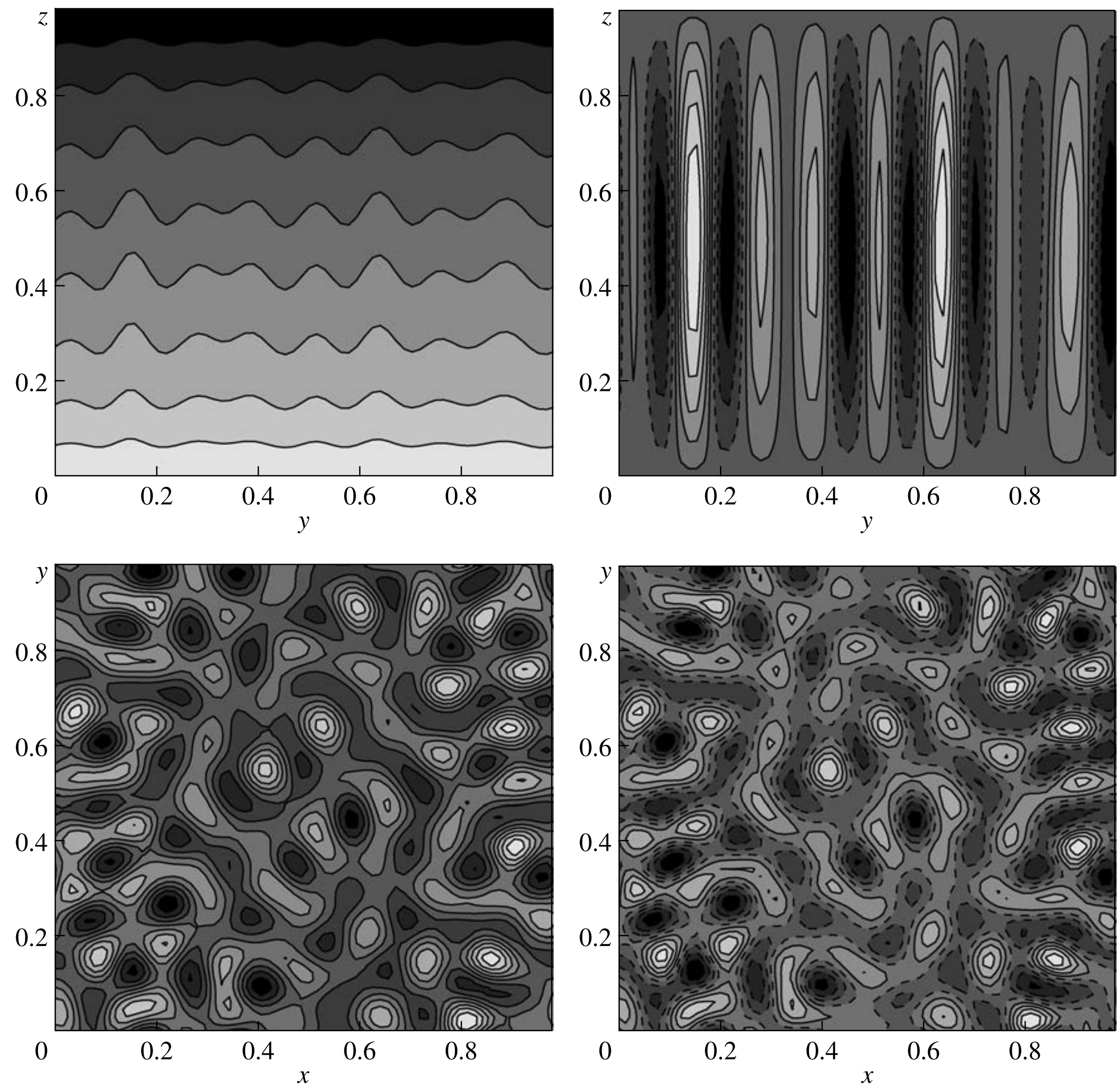

Fig. 3. Sections of the temperature field T and the vertical component of velocity V_z taking the rotation into account (regime R1). The upper row is for $x = 4.3$, the lower row is for $z = 0.8$. The field ranges are (0, 1), (–88, 127) and (0.17, 0.23), (–55, 86).

numbers the reverse cascade of energy $T_K^< > 0$ is observed. At the same time, in the larger part of the domain of wave numbers $0 < k < k_c$, the energy cascade is direct as before, $T_K^< < 0$.

THE LOCALITY OF ENERGY FLUX

Let us proceed to a question concerning the structure of triadic interactions. The diagram of T_{uu} fluxes for the regimes examined above is presented in Fig. 5. For the NR regime, the symmetry of the plot relative to diagonal $K = Q$ is clearly outlined. Analogous properties of T_{uu} are observed for the problem of 3D convection with free decaying [Debliquy et al., 2005] and with random induced force [Alexakis et al., 2007]. On the whole, the harmonics with $K > Q$ consume the energy in harmonics with $K < Q$ (the direct cascade of energy). The maximum of energy flux falls at the harmonics close to the diagonal harmonics with $K \sim Q$, i.e., the local energy transfer is present. Attention should be drawn to the existence of the domains (for example, $Q = 5$, $K = 20$), for which the nonlocal reverse cascade of energy is also observed (see discussion in the introduction). However, as a whole, the picture is close to the idealized Kolmogorov scenario. It is convenient to represent the diagram, given in Fig. 5 in the form of an integral over Q and K, as a function of K vs. Q (Fig. 6). The plot distinctly demonstrates the presence of the direct cascade, and also the local interaction and the local energy transfer.

Rotation changes the behavior of T_{uu} for $k < k_c$, leaving it practically without change for the high-frequency range

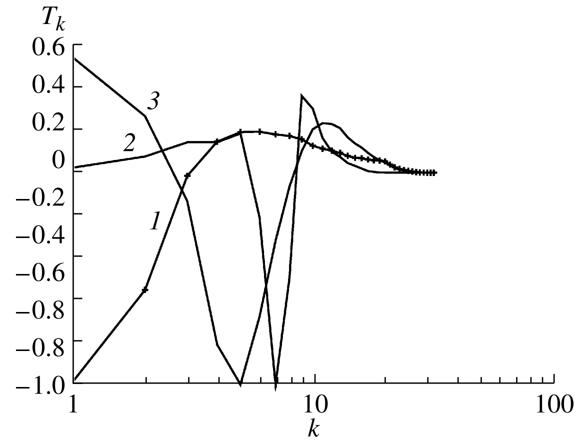

Fig. 4. Kinetic energy fluxes $T_K(k)$ for regimes (*1*) NR, (*2*) R1, and (*3*) R2.

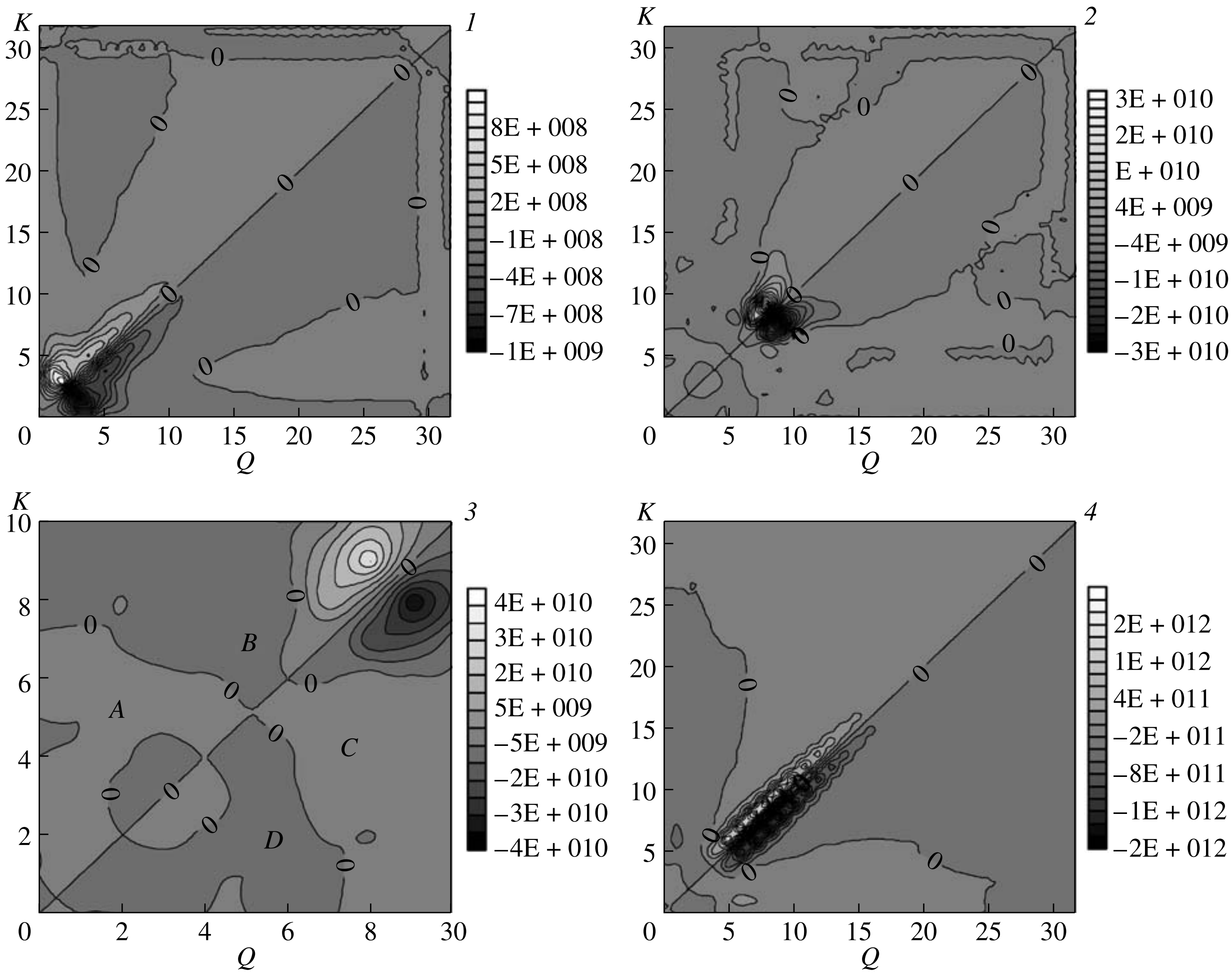

Fig. 5. Kinetic energy fluxes $T_{uu}(k)$ for regimes (1) NR, (2)–(3) R1, and (4) R2.

$k > k_c$. Let us examine the changes that appeared in more detail. The position of absolute maximums (minimums) falls to the region, close to k_c, i.e., energy is transferred to the region of large k from the leading mode. On the other hand, for $k < k_c$ a sharp decrease of the energy flux is observed, which corresponds to the approach of the system to the statistical equilibrium, which is also observed in Fig. 4. In the scaled-up version (see Fig. 5(*3*)) a finer structure for small k is observed: domain A with the direct cascade of energy, but with the equally probable energy transfer both from small $Q \sim K$ and from comparatively large $Q \sim 4K$. Domain B with the reverse cascade of energy (and also, as in domain A with the small amplitude of the energy flux of the order of 1/10th of the absolute maximum) has the extended strip-like domain form from $Q \sim K$ to $Q \sim 10K$. In Fig. 6 this corresponds to the appearance of the small negative minimum for $K > Q_2$.[2]

An increase in Ra (regime R2) leads to the shift of the domain with the reverse cascade of energy towards small Q and $K > Q$. As before, it is possible to speak about the existence of the state close to the uniform distribution for $k < k_c$. This regime is also characterized by the longer interval in the domain $k > k_c$ with the local energy transfer and with the direct cascade. The relative contribution of the domain with the reverse cascade becomes smaller (see Fig. 6(*3*)). At the same time, this contribution is transferred to the large-scale domain $k \ll k_c$, that can be of interest for problems of geodynamo, in which $k_c \sim 10^5$ and the domain of magnetic field generation (in the up-to-date estimates of the Reynolds magnetic number $R_m \sim 10^2–10^3$ the domain of magnetic field generation falls within the range $k \sim 1–10^3$) differ by at least several orders of magnitude.

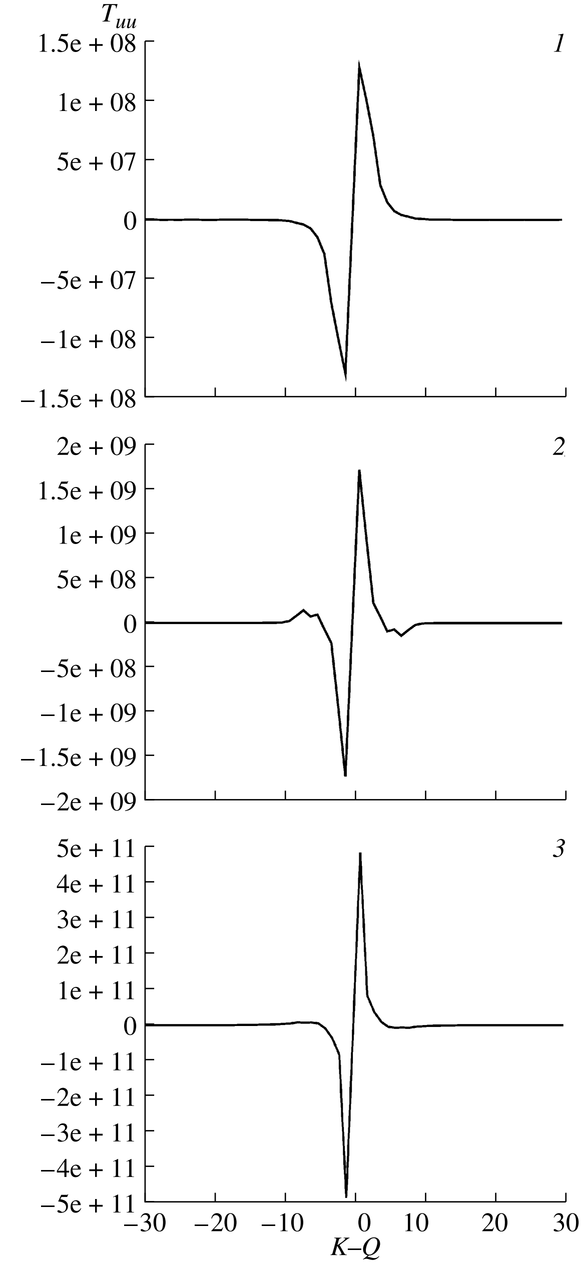

Fig. 6. Kinetic energy fluxes $T_{uu}(K - Q)$ for regimes (1) NR, (2) R1, and (3) R2.

DISCUSSION AND CONCLUSIONS

The problem examined above about the mechanism of the kinetic energy transfer in the fast-rotating liquid is one of the elements of the full planetary dynamo problem. The existence of the additional scale $\sim k_c^{-1}$, connected with the cyclonic convection significantly complicates the picture of energy transfer from one scale to the other. If in the Kolmogorov turbulence between the main scale (L) and dissipative scale ($Re^{-3/4}L$) the system remained completely self-similar with the direct kinetic energy flux over the spectrum, then with the appearance of rotation the possibility of the deceleration of energy transfer in the long-wave spectrum range ($k < k_c$) appears. In this range, the domains with the reverse cascade of energy appear, and energy is transferred directly from the small scales (with $k > k_c$). The latter resembles the idealized picture of the α-effect in the theory of the mean magnetic field, in which it is assumed that the magnetic energy of the large-scale magnetic field is created by the correlated pulsations of the small-scale velocity field. More consistent analysis (which is close to the analysis carried out above for T_{uu} fluxes) also indicates the appearance of nonlocal energy transfer [Verma, 2004; Alexakis et al., 2007].

In order to estimate the applicability of the obtained results to the conditions of the Earth's core it can be assumed that in the liquid Earth's core the Rayleigh number is larger than its critical value by a factor of 500 [Jones, 2000]. Since this value is higher than the value of Ra used by us in the R2 regime, it can be assumed that the spectrum of kinetic energy is close to the Kolmogorov one: $V = V_0 k^{-1/3}$, where $V_0 = 4 \times 10^{-4}$ m/s is the rate of the western drift of the magnetic field. Then, the ratio of the nonlinear term to the Coriolis force in the Navier–Stokes equation will be $\Re = \hat{Ro} k^{2/3}$, where $\hat{Ro} = \frac{V}{L\Omega} = 2 \times 10^{-6}$ is the dynamic Rossby number for the Earth. For $k_c^{-1} = E^{-1/3} = 10^5$, $\Re(k_c) = 3 \times 10^{-3}$, i.e., within the entire range of wave numbers $1 < k < k_c$ the geo-

[2] There is no comment on the behavior of the curve for K < Q (domains *C* and *D*), since this dependence is antisymmetric.

strophic balance of forces is present, and it is possible to speak about the applicability of the analysis carried out above and the conclusions in favor of the nonlocal kinetic energy transfer at all scales, where the existence of geomagnetic fields in the Earth's core is assumed.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, project no. 03-05-64074.

REFERENCES

1. A. Alexakis, P. D. Mininni, and A. Pouquet, "Shell to Shell Energy Transfer in MHD. I. Steady State Turbulence," Phys. Rev., **V.E 72** 046301–046309 (2005).
2. A. Alexakis, P. D. Mininni, and A. Pouquet, "Turbulent Cascades, Transfer, and Scale Interactions in Magnetohydrodynamics," New Journal of Pysics, **298** (9), l–20 (2007).
3. G. K. Batchelor, *The Theory of Homogeneous Turbulence* (Cambridge University Press, Cambridge, 1953) pp. 1–195.
4. F. H. Busse, "Thermal Instabilities in Rapidly Rotating Systems," J. Fluid Mech., **44**, 441–460 (1970).
5. F. Cattaneo, O. Emonet, and N. Weis, "On the Interaction between Convection and Magnetic Fields," Ap. J., **588**, 1183–1198 (2003).
6. S. Chandrasekhar, *Hydrodynamics and Hydromagnetic Stability*, (Dover Publications. Inc., New York, 1981), pp. 1–654.
7. O. Debliquy, M. Verma, and D. Carati, "Energy Fluxes and Shell-to-Shell Transfers in Three-Dimensional Decaying Magnetohydrodynamic Turbulence," Physics of Plasmas, **12**, 042309-1–042309-10 (2005).
8. U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press., Cambridge, 1995) pp. 1–296.
9. C. A. Jones, "Convection-Driven Geodynamo Models," Phil. Trans. R. Soc. London. **A358** 873–897 (2000).
10. C. A. Jones, P. H. Roberts, "Convection Driven Dynamos in a Rotating Plane Layer," J. Fluid Mechanics, **404**, 311–343 (2000).
11. R. H. Kraichnan and D. Montgomery, "Two-Dimensional Turbulence," Rep. Prog. Phys. **43**, 547–619 (1980).
12. M. Meneguzzi and A. Pouquet, "Turbulent Dynamos Driven by Convection," J. Fluid Mech., **205**, 297–318 (1989).
13. S. A. Orszag, "Numerical Simulation of Incompressible Flows within Simple Boundaries. I. Galerkin (Spectral) Representations," Stud. Appl. Math., **L** (51), 293–327 (1971).
14. J. Pedlosky, *Geophysical Fluid Dynamics* (Springer-Verlag, New York, 1987) pp. 1–710.
15. M. Yu. Reshetnyak, "Cascade Processes in Magnetostrophic Turbulence," Dokl. RAN (Geophysics), **420** (4) 527–531 (2008a).
16. M. Yu. Reshetnyak, "Heat Convection and Dynamo with the Fast Rotation Motion," Fiz. Zemli. **8**, 23–32 (2007).
17. M. Yu. Reshetnyak, "Some Properties of Cyclonic Turbulence in the Liquid Earth Core," Geomagnetism and Aeronomy, **48** (3) 416–423 (2008b).
18. M. Reshetnyak and B. Steffen, "Shell Models in Rapidly Rotating Dynamo Systems," Numerical Methods and Programming, **7** 85–92 (2006). (http://www.srcc.msu.su/ nummeth/english/index.html)
19. H. A. Rose and P. I. Sulem, "Fully Developed Turbulence and Statistical Mechanics," J. Physique., **39**, 441–484 (1978).
20. P. Tabeling, "Two-Dimensional Turbulence: a Physicist Approach," Phys. Reports, **362** 1–62 (2002).