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Dynamo Catastrophe, or Why the Geomagnetic Field Is So Long-Lived

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Abstract—From a linear analysis of the thermal convection model it follows that the scale of flows in the directions perpendicular to the Earth rotation axis is about 10^2 m when nonlinear interactions and a large-scale magnetic field are absent in the presence of thermal sources in the Earth's interior. Flows of such scales cannot generate a magnetic field because of a high ohmic dissipation. In this case, when the magnetic field decays, the geodynamo changes into the state when the generation of a magnetic field is terminated. This contradiction between theoretical assumptions and available paleomagnetic data is explained in the present work. It is indicated that this phenomenon is related to magnetic field inversions and excursions.

1. INTRODUCTION

The geomagnetic field has existed for at least 2 Gyr. Its existence is related to the dynamo processes in the Earth's liquid core. Radioactive heating and matter differentiation are the sources of energy. Up-to-date large-scale three-dimensional dynamo models describe the processes of heat transfer, hydrodynamics, and generation of the magnetic field. The models reproduce a wide spatial–temporal spectrum of the magnetic field, which agrees with the available archeological and paleomagnetic observations [Kono and Roberts, 2002]. It is time to comprehensively compare the models with observations and to analyze in detail used parameters.

Recent paleomagnetic observations [Langereis *et al.*, 1997] indicate that many excursions have existed beginning from the last magnetic field inversion (0.7 Myr ago); therefore, the developed geodynamo models should meet additional requirements. The excursion timescales are $(2–5) \times 10^3$ years. Thus, the geomagnetic field spends up to 20% of its time in a weak, non-dipole state [Lund *et al.*, 1998]. Based on this circumstance, Gubbins [1999] drew the conclusion that a timescale of about 500 year exists, which coincides with typical convective times in the Earth's liquid core and exceeds the diffusion time of the magnetic field in the solid core ($\sim 3 \times 10^3$ years). It is interesting that the existence of the characteristics processes with times of $\sim 10^2$ years and smaller in the entire liquid core was predicted long before the first observations [Roberts, 1965; Busse, 1970]. These observations and the gained numerical simulation experience required further studies. In this connection, Jones [2000] and Zhang and Gubbins [2000] considered the dynamo paradox, according to which the steady state of a magnetic field in rapidly rotating bodies, specifically in the Earth and other planets of the solar system as a whole, is doubtful. This phenomenon was called the dynamo catastrophe. The essence of this phenomenon consists in that, with decreasing magnetic field, flows in the Earth's liquid core can become small-scale in the plane perpendicular to the Earth rotation axis, energy loss by ohmic dissipation will increase, and the following generation of the magnetic field will become impossible. On the other hand, the magnetic field lifetime (10^9 years) is several orders of magnitude longer than not only the convective timescales in the liquid core (about 10^1–10^3 years) but also the magnetic timescales ($\sim 10^4$ years), which makes the above scenario doubtful. Below, we will try to explain this contradiction.

2. DYNAMO EQUATIONS AND BIFURCATION ANALYSIS

Let us consider the dynamo equations in the Earth's liquid core (see, e.g., [Jones, 2000; Kono and Roberts, 2002]) for an incompressible fluid ($\nabla \cdot \mathbf{V} = 0$) in a Boussinesq approximation in a layer rotating about the z axis in the (x, y, z) coordinate system at an angular velocity Ω. Having introduced the following units of measurement for velocity $\mathbf{V}$, time t, pressure P, and magnetic field $\mathbf{B}$: κ/L, L^2/κ, $\rho\kappa^2/L^2$, and $\sqrt{2\Omega\rho\kappa\mu_0}$ (where L is the unit of length, κ is the coefficient of molecular heat conductivity, ρ is the matter density, and μ_0 is the permeability of vacuum), we write the equations in the form

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{V} + q^{-1}\Delta\mathbf{B}$$

$$E\mathrm{Pr}^{-1}\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V}\right]$$

$$= -(\mathbf{B} \cdot \nabla)\mathbf{B} - \nabla\left(P + \frac{B^2}{2}\right) - \mathbf{1_z} \times \mathbf{V} + \mathrm{Ra}Tz\mathbf{1_z} + E\Delta\mathbf{V} \quad (1)$$

$$\frac{\partial T}{\partial t} + (\mathbf{V}\cdot\nabla)(T+T_0) = \Delta T.$$

Here T is the temperature disturbance relative to the equilibrium distribution $T_0 = 1 - z$. The dimensionless Prandtl, Ekman, Rayleigh, and Roberts numbers are specified in the form: $\mathrm{Pf} = \frac{\nu}{\kappa}$, $E = \frac{\nu}{2\Omega L^2}$, $\mathrm{Ra} = \frac{\alpha g_0 \delta T L}{2\Omega\kappa}$, and $q = \frac{\eta}{\kappa}$, where ν is the coefficient of kinematic viscosity, α is the coefficient of volumetric expansion, g_0 is the gravitational acceleration, δT is the unit of temperature, and η is the coefficient of magnetic diffusion.

Before we begin to analyze the complete dynamo system, let us consider the properties of thermal convection at a convection origination threshold, when the Rayleigh number (Ra) is near its critical value (Ra^{cr}). For this purpose, we reject all terms including magnetic field and the convective terms quadratic with respect to small fields. Following [Boubnov and Golitsyn, 1995], we eliminate pressure from (1) by applying the curl and curl–curl procedures to the Navier–Stokes equation and obtain:

$$E\mathrm{Pr}^{-1}\frac{\partial\nabla^2 V_z}{\partial t} = E\nabla^4 V_z + \mathrm{Ra}\Delta_1 T - \frac{\partial\Xi}{\partial z}$$
$$E\mathrm{Pr}^{-1}\frac{\partial\Xi}{\partial t} = E\nabla^2\Xi + \frac{\partial V_z}{\partial z} \qquad (2)$$
$$\frac{\partial T}{\partial t} + (\mathbf{V}\cdot\nabla)T_0 = \Delta T,$$

where $\Xi = \mathrm{rot}_z\mathbf{V}$, $\Delta_1 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$.

The substitution of $(V_z, \Xi, T) = (v_z, \xi, \Theta) \times e^{\gamma t + i(\omega t + k_x x + k_y y + k_z z)}$ into (2) results in

$$-E\mathrm{Pr}^{-1}(\gamma + i\omega)k^2 v_z = Ek^4 v_z - \mathrm{Ra}\Theta k^2 - i\xi$$
$$E(\mathrm{Pr}^{-1}(\gamma + i\omega) + k^2)\xi = i v_z \qquad (3)$$
$$(\gamma + i\omega + k^2)\Theta = v_z,$$

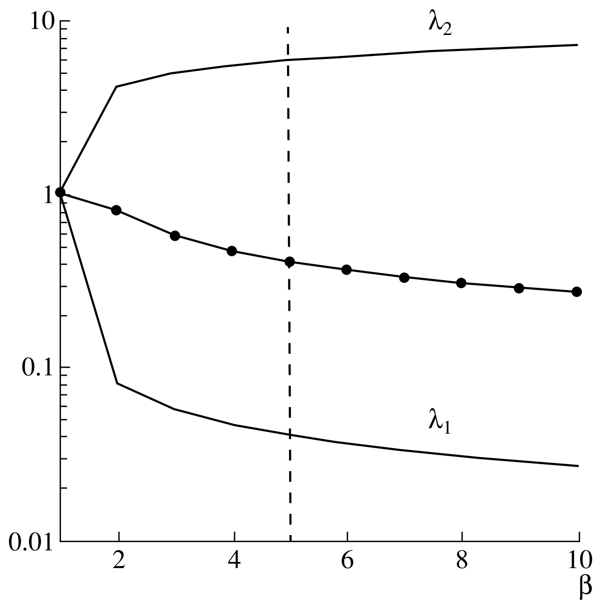
where $\sqrt{2}\,k = k_x = k_y \gg k_z \sim 1$, and from the divergence-free condition $(v_i k_i)$ it follows that $v_x \simeq -v_y$. The matrix form of (3) is $\mathbf{A}\cdot(v_z, \xi, \Theta)^T = 0$, where

$$\mathbf{A} = \begin{pmatrix} -E(\mathrm{Pr}^{-1}(\gamma + i\omega) + k^2) & \frac{i}{k^2} & \mathrm{Ra} \\ -i & E(\mathrm{Pr}^{-1}(\gamma + i\omega) + k^2) & 0 \\ -1 & 0 & \gamma + i\omega + k^2 \end{pmatrix}. \qquad (4)$$

The condition of $\mathrm{Det}\mathbf{A} = 0$ solvability at the generation threshold ($\gamma = 0$) gives: $\omega_{1,2,3} = 0, \pm\mathrm{Pr}^{1/2}k^{-1}E^{-1}(-E\mathrm{Ra}k^2 + 2E^2k^6 + E^2k^6\mathrm{Pr} + \mathrm{Pr})^{1/2}$, and $\mathrm{Ra}_1 = \frac{E^2k^6 + 1}{Ek^2}$. From the condition $\frac{d\mathrm{Ra}_1}{dk} = 0$ we have $k_1^{cr} = 2^{-1/6}E^{-1/3}$, $\mathrm{Ra}_1^{cr} = 3 \times 2^{-2/3}E^{-1/3}$. Similarly, $\mathrm{Ra}_2 = 2\frac{E^2k^6(\mathrm{Pr}+1)^2 + \mathrm{Pr}^2}{Ek^2(\mathrm{Pr}+1)}$ and $k_2^{cr} = 2^{-1/6}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}+1}\right)^{1/3}E^{-1/3}$, $\mathrm{Ra}_2^{cr} = 3 \times 2^{1/3}\mathrm{Pr}^{4/3}(\mathrm{Pr} + 1)^{-1/3}E^{-1/3}$, $\omega_2^{cr} = 2^{-1/3}\left(\frac{\mathrm{Pr}}{\mathrm{Pr}+1}\right)^{2/3}(2 - 3\mathrm{Pr}^2)^{1/2}E^{-2/3}$ (the condition $\frac{d\mathrm{Ra}_3}{dk} = 0$ gives the same results). Since $E = 10^{-15}$ and $\mathrm{Pr} = 0.1$ for the Earth's core, the ratio of the longitudinal and transverse scales at the generation threshold is about $E^{-1/3}$, i.e., is very large.

Assume that $\mathrm{Ra} = \beta\mathrm{Ra}^{cr}$, where $\beta > 1$. Let us determine k at which the growth rate of eigenfunctions (γ) is positive. For simplicity, we first consider the solutions with $\omega = 0$. The compatibility condition gives the relationship $k^6E^2 = E\mathrm{Ra}k^2 - 1$. At $\beta > 1$, there exists the interval of wavenumbers $\Lambda = (\lambda_1 k^{cr}, \lambda_2 k^{cr})$ with $0 < \lambda_1 < 1$, $\lambda_2 > 1$ where $\gamma > 0$. For $E = 10^{-15}$, the behavior of $\lambda_1(\beta)$ and $\lambda_2(\beta)$ is shown in figure, which also illustrates the $\lambda_1(\beta)$ and $\lambda_2(\beta)$ dependences for the second branch of solutions with $\omega_2 \neq 0$ and $\mathrm{Pr} = 0.1$. (For large wavenumbers, i.e., for λ_2, the graphs coincide on the selected scale.)

Thus, all growing modes, except nonlinear terms, are located in the Λ range of wavenumbers. We now consider the behavior of $\lambda_1(\beta)$ and $\lambda_2(\beta)$ at $\beta \gg 1$. From the compatibility condition it follows that $\lambda_2 \sim \beta^{1/4}$ for $k \gg k^{cr}$ and $\lambda_1 \sim \beta^{-1/2}$ for $k \ll k^{cr}$. We also note that convection is generated on the nominal scale at $\mathrm{Ra} \sim E^{-1}$! Figure indicates that the value $\beta = 500$ taken for the Earth [Jones, 2000] corresponds to expansion of the



Expansion of the generation region with increasing Rayleigh number for $E = 10^{-15}$ and Pr = 0.1. The upper branch of λ_2 is identical for $\mathrm{Ra}_1^{\mathrm{cr}}$, k_1^{cr} and $\mathrm{Ra}_2^{\mathrm{cr}}$, k_2^{cr}. The dependence for λ_1 is different. Circles correspond to $\mathrm{Ra}_2^{\mathrm{cr}}$ and k_2^{cr}. The dashed line marks the regime in the Earth's core at $\beta = 5$ [Jones, 2000].

generation region with $\lambda_1 \sim 5 \times 10^{-2}$ and 3×10^{-1} and $\lambda_2 \sim 5$. The scale corresponding to λ_1 is equal to $l_{\lambda_1} = \frac{L}{\lambda_1 k_1^{\mathrm{cr}}} = \frac{3.6 \times 10^6}{0.05 \times 9 \times 10^4} \sim 800$ m (or $l_{\lambda_1} = \frac{L}{\lambda_1 k_2^{\mathrm{cr}}} = \frac{3.6 \times 10^6}{0.3 \times 4 \times 10^4} \sim 300$ m in the second case with $\omega \neq 0$); i.e., $l_{\lambda_1} \sim (10^{-3}\text{–}10^{-4})L$. When the magnetic Reynolds number is estimated based on the nominal scale (L) and westward drift velocity ($V_{WD} = 0.2°$ per year), $R_m = \frac{V_{WD}L}{\eta} \sim 10^3$ ($\eta = 1$ m^2 s^{-1}). Only in the most optimistic estimations, the magnetic Reynolds number will be about unity on this scale: $r_m = (10^{-3}\text{–}10^{-4})R_m \lesssim 1$. Note that this does not contradict the observations of the magnetic field on the liquid core surface: according to [Lowes, 1974], the magnetic field spectrum for $2 < k < 14$ decreases only as $\sim e^{-0.1k}$. If we assume that the generation of a large-scale magnetic field due to large-scale convection is explained by an increase in the scales by a magnetic field, then the convection scale decreases with decreasing magnetic field and further generation of a magnetic field terminates. Based on the above arguments, Jones [2000] and Zhang and Gubbins [2000] concluded that the geodynamo is unstable. Since the magnetic field repeatedly changed its polarity and, generally speaking, could become several orders of magnitude weaker at least locally, it becomes unclear why the geomagnetic field exists so long. Below, we will consider the mechanism that makes it possible to limit the instability in the liquid core and to explain the existence of the geomagnetic field on geological timescales.

3. ENERGY BALANCE. DISCUSSION

Let us consider in more detail the transformation of the energy, which comes from thermal sources, in the liquid core at Ra > Ra$^{\mathrm{cr}}$. Assume that all energy coming from thermal sources changes into the kinetic energy. In such a case, the kinetic energy is determined from the balance between the Archimedean force work $A = \mathrm{Ra}TV \sim \mathrm{RaRePr}$ and viscous dissipation $D_\nu \sim EV^2 = E(\mathrm{RePr})^2$. Assuming that Pr = 0.1, $\mathrm{Re} = \frac{V_{WD}L}{\nu} = 10^9$ ($\nu = 10^{-6}$ m^2 s^{-1}), $E = 10^{-15}$, and Ra > 10^6 [Jones, 2000; Gubbins, 2001], we obtain that $A = 10^{14}$ and $D_\nu = 10$ (i.e., $A \gg D_\nu$), which is a contradiction: viscous dissipation cannot compensate thermal energy input.

Assume that all energy coming form thermal sources changes into the magnetic energy. In this case the energy loss by viscous dissipation is negligible ($D_\nu = 10$), and convection is merely a transfer mechanism that cannot accumulate energy. The entire energy budget is controlled by a magnetic field amplitude and ohmic dissipation. From the equality of the Archimedean force work and loss by ohmic dissipation we have: $A \sim B^2$ or $B \sim (\mathrm{RaRePr})^{1/2} = 10^7$, which corresponds to 10^{-2} T and is compatible with the observations and numerical simulation. This variant is usually accepted a fortiori; however, it is still unclear what will happen if any factor causes a decrease in the magnetic field. As was mentioned above, such a magnetic field weakening can lead to a decrease in the scale and to a termination of magnetic field generation. Disappearance of a magnetic field will, in turn, result in an uncompensated growth of kinetic energy: in this case all thermal energy will change into viscous dissipation. To estimate a rate amplitude, it will be important to take into account that a viscosity coefficient depends on this rate, i.e., to introduce turbulent viscosity.

We begin with a standard model of mixing. The turbulent Ekman number is $E^T = E(1 + C_1V) = E(1 + C_1\mathrm{PrRe})$, where $C_1 = 0.1$ is the mixing constant. Then, $A = 10^{14}$ as before, and the loss by viscous dissipation is $D_\nu = C_1EV^3 = C_1E(\mathrm{PrRe})^3 = 10^8$. It turns out that viscous dissipation cannot compensate the Archimedean term even if the introduced nonlinearity is taken into account. Before discussing more intricate models, we refer to one circumstance of prime importance. In a model of mixing, the kinetic energy increases by four orders of magnitude, or the magnetic Reynolds number

(R_m) increases by a factor of 100 ($R_m = 10^5$). In such a case, $r_m \sim 10^2$, and the magnetic field can be generated on scales with $\gamma > 0$. In other words, a decrease in the magnetic field results in an increase in the α effect (this effect is considered in more detail in [Vainshtein *et al.*, 1980]). *Such a feedback can fully prevent the magnetic field from disappearing.* We will see below that this is not a single stabilizing factor.

The above model of mixing is based on the ideas of a direct Kolmogorov–Obukhov energy cascade, when energy is transferred from large scales to small ones. A rapid rotation can substantially change the situation. As was indicated in the previous section, $\gamma > 0$ only in a small range of transverse wavenumbers (Λ). In other words, all flows with characteristic scales in the azimuthal direction available from geomagnetic observations ($k \le 14$) are products of nonlinear interactions and are related to an inverse energy cascade. This phenomenon can be explained in two ways. The first mechanism is purely thermal hydrodynamic, ignoring magnetic field. This model assumes the existence of an inverse energy cascade. On the whole, the energy cycle is as follows: dissipation proceeds on scales close to $\sim E^{1/3}$ and increases turbulent viscosity by a factor of $\sim E^{-1/3}$ [Reshetnyak and Steffan, 2004; Reshetnyak, 2004]. Through the inverse energy cascade, energy is transferred to large scales in the transverse direction with respect to k_x; from these scales, energy returns in the dissipation region through the direct cascade in the k_z direction. Note that the presence of the inverse energy cascade in two-dimensional turbulence is a well-known fact. (see, e.g., [Tabeling, 2002]). This cascade is also successfully reproduced with the help of cascade models of turbulence for spaces with an intermediate dimensionality ($2 < n < 3$) [Aurell *et al.*, 1994]. The energy spectrum slope varies from the Kolmogorov (–5/3) to white. We can qualitatively trace a direct cascade blocking by rotation based on the following assumptions [Vainshtein *et al.*, 1980]. Let us represent the nonlinear term in the form $(\mathbf{V} \cdot \nabla)\,\mathbf{V} = \nabla(V^2/2) - \mathbf{V} \times (\nabla \times \mathbf{V})$. We can eliminate the gradient term by inserting this term into pressure. In the presence of rotation, turbulence becomes gyrotropic (the average helicity $\langle \mathbf{V} \cdot (\nabla \times \mathbf{V})\rangle$ is nonzero). In such a case, the term including curl starts statistically tending to zero, and the energy transfer terminates. One more mechanism exists in addition to nonlinear blocking: for λ_1, the last two terms in the first equation in (2) anticorrelate and block the energy income into the system.

The presence of a magnetic field makes the situation substantially more intricate. First, the critical Rayleigh number is very sensitive to a magnetic field, and $\mathrm{Ra}^{cr} \sim E$ in the magnetostrophic regime [Zhang and Gubbins, 2000]; i.e., a strong magnetic field can cause an additional type of instability: spontaneous intensification of convection. Second, an inverse magnetic energy cascade can exist together with a hydrodynamic inverse energy cascade. Parametrization of this phenomenon is nothing but the α effect. Taking into account a growth of kinetic energy during magnetic field damping (see above), we can state that this cascade can also cause the observed large-scale fields. Note that, within the scope of this approach, the loss by turbulent dissipation for the Navier–Stokes equation and the ohmic loss can be of the same order of magnitude; therefore, a disappearance of the magnetic field will not result in a serious change in the energy budget of the Earth's core. However, such phenomena as geomagnetic field excursions and inversions can be closely related to the appearance of local (for excursions) or global (for inversions, in the entire volume of the liquid core) regions where the energy contribution of small-scale flows is larger than that of a standard geomagnetic field. Since the characteristic times of inversions and excursions is close to the time of development of hydrodynamic instabilities ($\sim 10^3$–10^4 years), the interruption of the dynamo mechanism, which results in a temporary weakening of the large-scale dipole constituent, can be related to the appearance of small-scale turbulence. In other words, inversions and excursions can be considered as failed dynamo catastrophes. This is confirmed by numerous results of numerical simulation, which demonstrate that the energy of small-scale fields increases during inversions, and is reflected in low-mode models of the geodynamo [Shalimov, 2003]. A more audacious statement consists in that the origination of instabilities can also result in the appearance of jerks, e.g., in the form of a short-term termination of an energy cascade. The magnetic field intensity on the nominal scale (L) cannot change during this period; nevertheless, a magnetic field cascade can be blocked on large k. In this case the core will be filled with hydrodynamic turbulence with a scale $\sim 1/k$, and an inverse energy cascade will stop for a certain time, which will be reflected in the behavior of higher time derivatives of the magnetic field. This phenomenon can be accompanied by an intensification of hydrodynamic turbulence with a characteristic time of about a year in the entire volume of the Earth's core.

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