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In Memory of G.N. Petrova

Geomagnetic Field Intensity and Suppression of Helicity in the Geodynamo

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Abstract—The observed intensity of the geomagnetic field is consistent with the concept of magnetostrophic balance and disagrees with the concept of geostrophic balance. However, models of astrophysical dynamos usually involve concepts that are closely related to the geostrophic balance. This difference is shown to be due to the relatively slow rotation of stars and galaxies, which are studied by astrophysicists. Within the bounds of mean-field dynamo theory, a simple model of helicity suppression making the two approaches mutually consistent is proposed.

1. INTRODUCTION

The late Galina Nikolaevna Petrova took a keen interest in the fundamental, qualitative aspects of geomagnetism. During the last few months of her life, she insistently proposed that we present at a seminar a paper analyzing the amount of consistency between the current theoretical concepts of the geodynamo and the general ideas of dynamo theory. Life has turned out in such a way that this problem has to be addressed in a paper dedicated to her memory.

The classical dynamo theory involves the concept of a mean, large-scale field, whose self-excitation is related to the mean helicity of convection and the differential rotation in the Earth's outer core. This scenario was independently proposed by S.I. Braginskii in the context of the geodynamo and by Parker, Steenbeck, Krause, and Radler in the context of astrophysical dynamos. Modern magnetic hydrodynamics can describe the dynamo on a microscopic level, without invoking mean values explicitly. In geodynamo theory, the possibility of microscopic description was demonstrated in the last decade by Glatzmaier and Roberts [1995].

Nevertheless, the possibility of a closed phenomenological description of the behavior of mean fields appears to be an attractive feature of the theory. Modern research on astrophysical dynamos has developed many techniques for parametrizing the helicity and other mean characteristics of convection (or turbulence); the microscopic approach was only utilized to substantiate these methods, which enable the construction of closed models describing the generation of the magnetic fields of stars and galaxies. As regards the geodynamo, such techniques are fewer. It is not clear beforehand whether the astrophysical parametrization methods of mean characteristics of convection are applicable to the geodynamo. In this paper, we show how this can be done for the law of helicity suppression.

2. GEOSTROPHIC AND MAGNETOSTROPHIC BALANCES

Compared to other types of dynamo, the geodynamo appears to be a highly specific phenomenon. In particular, the Earth (and many planets) has a very high velocity of rotation and a very large magnetic energy density (compared to the convection energy).

Dynamo theory begins with an examination of the exponential increase of an initially small magnetic field. As the field increases, so does the Lorentz force, which slows down the increase in the magnetic field. The specific estimate of the steady-state value of the magnetic field that is attained through the action of the dynamo depends on the particular force with which the Lorentz force is compared. Astrophysical applications of dynamo theory are usually based on comparing the Lorentz force with the inertial forces. The Coriolis force is assumed to be equilibrated by the pressure gradient. This approximation, which is referred to as the geostrophic balance in geomagnetism, gives

$$(H\nabla)H/4\pi \sim \rho(V\nabla)V, \qquad (1)$$

where H is the magnetic field, V is the velocity, and ρ is the density in the outer core. Assuming that the spatial scales of the magnetic field and the velocity field are comparable, we obtain the approximate equality of the densities of the kinetic (E_K) and the magnetic (E_M) energies [Busse, 1976; Jacobs, 1979]

$$\frac{H^2}{8\pi} \sim \frac{\rho V^2}{2}. \tag{2}$$

Note that the kinetic energy means here the kinetic energy of convective motions and differential rotation, because solid-body rotation does not directly lead to magnetic field generation but affects it indirectly by producing helicity under the action of the Coriolis force.

This simple estimation is effective in astrophysics but encounters difficulties when applied to the geodynamo. Actually, given the westward drift rate V_{WD} = 0.04 cm/s and the liquid core density ρ = 10 g/cm^3, we obtain $E_K = 8 \times 10^{-3}$ erg/cm^3 for the velocity of differential rotation. This same value gives the upper bound on the convection energy density, because otherwise the differential rotation would be indistinguishable against the background of convective motions. Estimates of this energy density in particular models of magnetic field generation give values somewhat smaller than this upper bound. For example, Anufriev *et al.* [1997] obtained $E_K = 5 \times 10^{-4}$ erg/cm^3.

Estimation of the geomagnetic field energy density is based on the directly observed value of the magnetic field on the surface (0.5 Gs). Extrapolation of the poloidal component of the magnetic field to the core–mantle boundary gives 5 Gs [Bloxham, 1987]. The toroidal component of the magnetic field is not directly observable due to the screening effect of the mantle, but it is undoubtedly not smaller than the poloidal component, which yields a lower bound of the magnetic energy density of about 1 erg/cm^3, i.e., about 100 times higher than the kinetic energy density. More realistic estimates of the magnetic energy density are model-dependent. For example, the well-known estimate of the toroidal field of 500 Gs [Hide and Roberts, 1979] gives E_M = 3×10^2 erg/cm^3, and the numerical model of Glatzmaier and Roberts [1995] gives $E_M/E_K = 4 \times 10^3$ erg/cm^3.

It is common knowledge that more realistic estimates of the magnetic energy density for the geodynamo and planetary dynamos are based on the magnetostrophic balance, in which the Coriolis force is counterbalanced by the Lorentz force (e.g., see [Eltayeb and Roberts, 1970; Curtis and Ness, 1986]):

$$(H\nabla)H/4\pi \sim 2\rho\Omega \times V, \tag{3}$$

where Ω is the angular velocity. Assuming that the spatial scales of the magnetic field and differential rotation are the same and coincide with the characteristic size of the outer core L, we obtain

$$E_M/E_K \sim \mathrm{Ro}^{-1}, \tag{4}$$

where $\mathrm{Ro} = V(2\Omega L)^{-1}$ is the Rossby number. With the Earth's value of $\mathrm{Ro} = 4 \times 10^{-7}$, this rough approximation even leads to an overestimated value of E_M/E_K. To fit the data of observations, the estimate of the magnetostrophic balance can be adjusted, for example, by introducing different scales for the angular velocity and the magnetic field that do not coincide with L; however, considering the large uncertainties in the estimated energy densities, this approach is unacceptable here. Beginning with the so-called Bode law proposed by Blackett [1947] (for its present-day version, see [Cain *et al.*, 1995]), a whole series of scaling laws have been proposed in the literature for planetary dynamos on the basis of considerations of this kind. In this context, mention can be made of the recent papers by Starchenko [1996, 1999].

For astrophysical dynamos, the Rossby number is usually not so small; therefore, even if the observations favor the estimation of the magnetostrophic balance (e.g., see [Baliunas *et al.*, 1996]), the differences between possible estimates are not so dramatic.

It is conventionally stated that the dynamo describes the kinetic-to-magnetic energy conversion. The estimates given above mean that this point of view should be substantially revised: differential rotation and convection are only mechanisms capable of converting the energy of the overall rotation, which cannot act directly as the dynamo, into the energy of the magnetic field. Generally speaking, the real velocity field can basically differ from the hypothetical velocity field that would have arisen with an initially small magnetic field.

Detailed models of planetary dynamos confirm the feasibility of the above scenario; however, they either consider the mean helicity as a preset quantity (models of a nearly axisymmetric dynamo; e.g., see [Gubbins and Roberts, 1987]) or entirely ignore this concept (Glatzmaier and Roberts' models). It is by no means obvious beforehand that the assumptions of these models can be made consistent with the ideas of helicity suppression developed for astrophysical dynamos.

3. HELICITY SUPPRESSION MODEL FOR THE GEODYNAMO

In the majority of astrophysical dynamo models, one does not need to consider how the magnetic field changes the flow and can restrict oneself to the naive idea that, as soon as the magnetic field energy becomes comparable to the kinetic energy, the mean helicity (the coefficient α in the model equations) decreases and any further increase in the magnetic field ceases. To be specific, we will hereafter refer to the helicity suppression model used by Brandenburg *et al.* [1989]:

$$\alpha = \alpha_0 f(E_M/E_K), \tag{5}$$

where α_0 is the hypothetical helicity value for a body with an initially small magnetic field and f is a decreasing function such as

$$f(E_M/E_K) = \frac{1}{1+E_M/E_K}. \qquad (6)$$

The limitation of this idea is obvious, but it leads to quite reasonable results for the Sun, stars, and galaxies, and more complex models account mainly for details of the process. However, the application of this idea to planetary dynamos should obviously stabilize the magnetic field at a level that does not exceed the value given by the estimate of the geostrophic balance.

Of course, one might insist that this helicity suppression scheme is too primitive to be applied to the geodynamo. However, we will show that, once the arguments leading to the model (5)–(6) are slightly adjusted, they become quite acceptable for the geodynamo. This can be demonstrated by examining how the initial helicity value α_0 is estimated.

According to the ideas of the electrodynamics of mean fields [Krause and Radler, 1980], the mean helicity of convection in a rotating body is related to the Coriolis force acting on a buoyant vortex. This leads to the estimate

$$\alpha = \frac{\Omega l^2}{L}, \qquad (7)$$

where l is the turbulence scale. This estimate has been actually used since mean-field dynamo theory began to be developed in the 1960s, but its role was realized much later. Therefore, it is difficult to give a precise reference to the paper in which it was originally proposed; traditionally, it has been ascribed to Krause.

Krause's estimate works well for astrophysical dynamos but yields an overestimated value of α_0 for the geodynamo. Actually, setting $l/L = 0.1$, we obtain $\alpha_0 =$ 35 cm/s. Since α_0 is measured in velocity units, it is natural to compare it with the westward drift velocity, which is as low as 0.04 cm/s. The geodynamo helicity estimate given by Krause diverges from the upper bound by the value of α proposed by Moffatt [1978]. Moffatt's estimate is based on the relationship between α_0 and the correlation properties of convection:

$$\alpha = \tau\langle v \operatorname{curl} v\rangle, \qquad (8)$$

where v is the convection velocity, $\tau = l/v$ is the characteristic time of vortex revolution, and $\langle\ldots\rangle$ is the sign of averaging over the ensemble of convective motions. Assuming that $\operatorname{curl} v \sim v/l$, we obtain

$$\alpha \le v \qquad (9)$$

(the sign $\le$ is placed because the velocity and the vortex are not necessarily perfectly correlated). In the case of the geodynamo, it is natural to relate the estimate of v to the westward drift velocity.

Thus, the Coriolis force could produce a much greater α effect in the Earth's outer core than is consistent with Moffat's estimate. In other words, the Coriolis force makes all vortices, say, right-handed and could give rise to many more such vortices rotating in the given direction if they really existed in the convection flow.

In order to match the assumptions of astrophysical dynamo models on the helicity suppression to the real geodynamo conditions, it is sufficient to assume that the helicity suppression model (5)–(6) relates only to Krause's estimate. In other words, the Lorentz force begins to suppress the production of helicity by the Coriolis force as soon as the energy densities of the magnetic field and motions become equal, but helicity is produced so efficiently that this suppression becomes effective only at a much higher magnetic energy. In this context, we propose the following simple formula as a law of helicity suppression in the geodynamo:

$$\begin{aligned}\alpha &= \min\left(v, \frac{\Omega l^2}{L} f(E_M/E_K)\right) \\ &= \alpha_0 \min(1, \mathrm{Ro}^{-1} f(E_M/E_K)),\end{aligned} \qquad (10)$$

where for simplicity we do not discriminate between the velocities of the westward drift, differential rotation, and convection, on the one hand, and set $l = L$, on the other hand; it is not difficult to apply corrections for the differences in these quantities.

The helicity model proposed above implies that the helicity suppression can be ignored up to magnetic field values corresponding to the estimated magnetostrophic balance $E_M/E_K = \mathrm{Ro}^{-1}$, after which helicity suppression participates in the magnetic field stabilization along with the variation in large-scale motions. Actually, this idea underlies the theory of a nearly axisymmetric dynamo and, in this sense, model (8) can be regarded as a means of making this theory consistent with the astrophysical dynamo concepts.

4. CONCLUSIONS

We have shown that the assumptions of the models of astrophysical dynamos and the geodynamo can be made mutually consistent, at least roughly, by taking into account specific features of the convection in the outer core due to the very rapid rotation of the Earth as a whole. Of course, more detailed models suppressing the geodynamo helicity require a careful examination of rapidly rotating convective media, which was initiated, in particular, by Rüdiger and Kichatinov [1993].

The simple formula proposed in this paper for the helicity suppression is essentially related to the estimation of convection velocities from the westward drift velocity. In other words, it solves the problem only partially, shifting the focus to why the Lorentz force does not increase the velocity of convective motions to the value corresponding to the estimate of the geostrophic balance. It is quite possible that, in reality, the westward drift velocity characterizes only the differential rotation velocity and one component of chaotic convective motions in the outer core, whereas the other, more intense component is related to the geostrophic balance and, for some reason, does not contribute to the dissipation of the large-scale magnetic field. It is likely that this intense component is represented by Alfven waves, whose ensemble itself produces the helicity.

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