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Wavelet Analysis of Geomagnetic Field Intensity for the Past 4000 Years

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Abstract—The wavelet time spectrum of the geomagnetic field intensity variation was constructed from archeomagnetic data for the past 4000 years by the example of series for Bulgaria, Georgia, and Central Asia. A 1750-year variation is confidently identified in all of the three series. The found phase shifts between the oscillations of this period in the three regions are related to a wave drifting from the east to the west at a rate of $v_\varphi \approx +0.2°$/yr. The correlation between the time series weakens as the characteristic time of the processes decreases. The character of the remaining oscillations in the wavelet spectra is discussed.

INTRODUCTION

One of the manifestations of the processes proceeding within the Earth's liquid core is secular variations (*SV*) in the geomagnetic field. Following the existing views [1, 2], the *SV* time spectrum is discrete, and the observed pattern of variations in the geomagnetic field intensity can be represented by a superposition of oscillations with periods from a few tens of years to a few thousands of years: 60, 100, 360, 500, 700, 1200, 1800, 2400, 3500, and 5000 years.

The basic methods used to determine the spectrum characteristics of field variations are the Fourier analysis and the maximum entropy method (MEM) [3]. Note that both these methods are based on the *a priori* assumption that the processes in question are strictly periodic. However, there is no direct reason to believe that the complex nonlinear system representing the Earth's liquid core exhibits a strictly periodic behavior. It is known that the use of the Fourier representation for approximately periodic signals may lead to the appearance of a large number of spurious peaks in the spectrum. Moreover, the analyzed series often contain gaps, which also complicate the Fourier analysis.

In this work, we use a relatively new method of spectral analysis termed wavelet analysis [4]. Wavelets are a set of oscillating self-similar functions of various scale that are localized in both the physical and Fourier spaces. The wavelet analysis allows one to study the processes with unsteady-state spectra, to follow the phase behavior of a quasiperiodic oscillation, and to estimate its energy parameters. The spectra, thus obtained, are more smooth, since the multiple and combination frequencies are suppressed in them (examples of the wavelet analysis can be found, for example, in a paper by Frick *et al.* [5]). The wavelet analysis can be generalized to time series with gaps in observations (adaptive wavelet analysis) [5, 6].

In this paper, from archeomagnetic data for the last 4000 years [7], we evaluated the spectrum characteristics of the SV field for three time series of the geomagnetic field in Bulgaria, Georgia, and Central Asia.

ARCHEOMAGNETIC DATA

Figure 1 presents the Eurasian data on the geomagnetic field intensity variations [7]. The time dependences of the field intensity are referred to the coordinates that are average values for each of the sampled region: Bulgaria (27°E, 42°N), Georgia (44°E, 42°N), and Central Asia (65°E, 40°N). The data are nonuniformly distributed over the time scale. The majority of the Georgian determinations goes back to the I millennium B.C., and that of the Bulgarian determinations are dated to 1000 A.C. The data for Central Asia are distributed more uniformly in time than the data for the other two regions. Note that only several tens of determinations in Bulgaria are known for II millennium B.C., while the number of determinations in Central Asia and Georgia is a few times greater. However, the number of A.D. determinations for Bulgaria is two and three times larger than that for Central Asia and Georgia, respectively.

The study of geomagnetic field intensity variations based on methods with the Fourier transformations [7–9] revealed the following sets of periods: 1560, 1135, 735, 585, 450, and 340 years for Bulgaria, 3570, 1770, 1000, 800, and 500 years for Georgia, and 3400, 1600, 1020, 680, 470, and 330 years for Central Asia. The variance

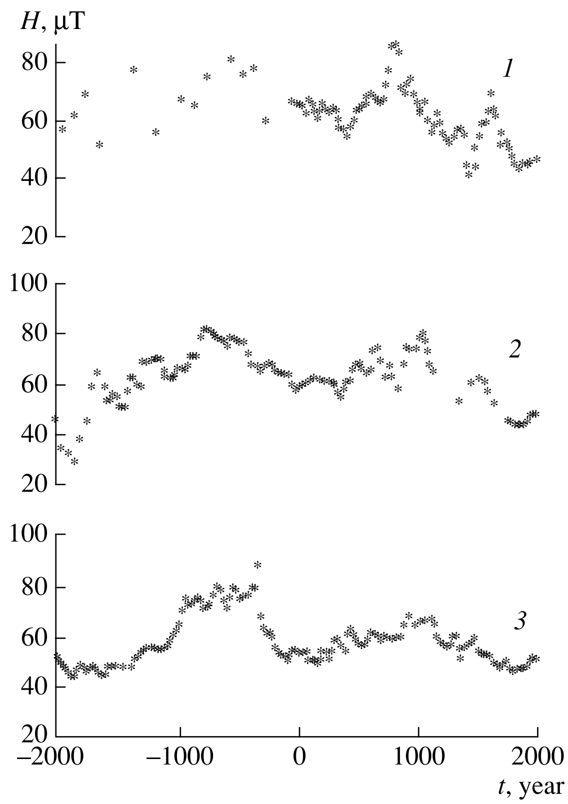

Fig. 1. Time variations in the geomagnetic field intensity in (*1*) Bulgaria, (*2*) Georgia, and (*3*) Central Asia.

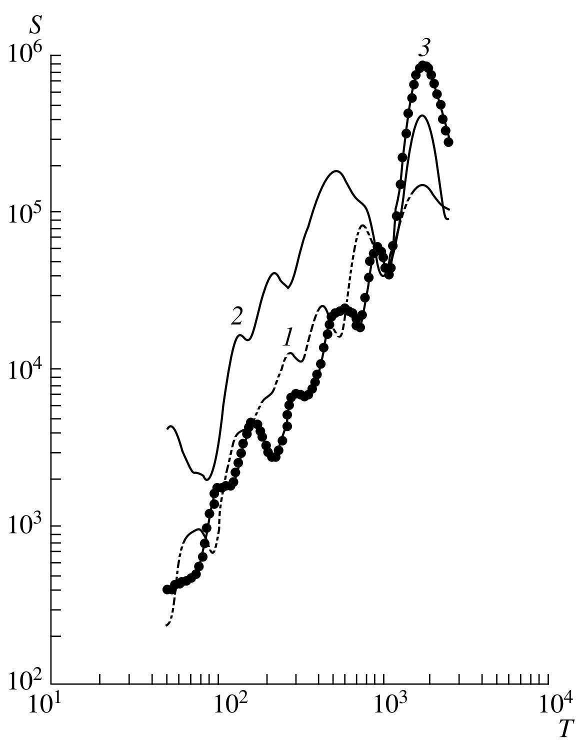

Fig. 2. Integral wavelet spectra for (*1*) Bulgaria, (*2*) Georgia, and (*3*) Central Asia.

in the estimated periods for the three regions does not exceed 15%, which appears to be comparable to errors of observations.

WAVELETS

The continuous wavelet transformation of a function $f(t)$ is defined by the quantity

$$w(a, t) = C_\psi^{-1/2} a^{-\sigma} \int_{-\infty}^{\infty} \psi\left(\frac{t' - t}{a}\right) f(t') dt',$$

where a is the parameter characterizing the timescale, $\psi(t)$ is a function called the analyzing wavelet, C_ψ is the normalizing constant, and σ is a normalizing coefficient (see below). The wavelet is a function with a small number of oscillations and the zero mean $\int_{-\infty}^{\infty} \psi(t)\, dt = 0$ (the condition providing the existence of the inverse transformation).

The choice of a particular wavelet form depends on the purpose of the analysis. We apply the high-resolution Morle wavelet $\psi(t) = e^{-t^2/2} e^{i2\pi\omega_0 t}$ to analyze the spectrum and wavelet $\psi(t) = (1 - t^2)e^{-t^2/2}$, usually named the “Mexican hat.”

The wavelet transformation recasts a function of one variable t into a plane of two variables t and α, where t characterizes the position of the wavelet center on the time axis, and parameter a controls the time scale of oscillations. For the Morle wavelet, this parameter coincides with the oscillation period. An analog of the Fourier spectrum is the so-called integral wavelet spectrum obtained by integrating the square of the wavelet modulus along the time axis for $\sigma = 1/2$:

$$S(a) = \int_{-\infty}^{\infty} |w(a, t)|^2 dt.$$

While calculating the wavelet transformation, the problem of edge effects arises, which becomes more acute for a greater timescale (i.e., for a shorter time series). Problems of the same type occur in considering the series, including gaps in data [5]. Below, we use an algorithm proposed by Galyagin and Frick [6] specifically for analyzing data with gaps. The idea of the method is to carry over the problem of gaps from analyzed function $f(t)$ to analyzed wavelet ψ. Considering that namely the wavelet is known in a restricted interval, we correct its form so as to retain its spectral properties. It is important that the method does not involve any interpolation of data.

RESULTS OF THE ANALYSIS

Figure 2 shows the integral wavelet spectra found with the help of the adaptive Morle wavelets. All three spectra are dominated by the peak at a period of a = 1750 ± 50 yr. The peak is especially pronounced in the

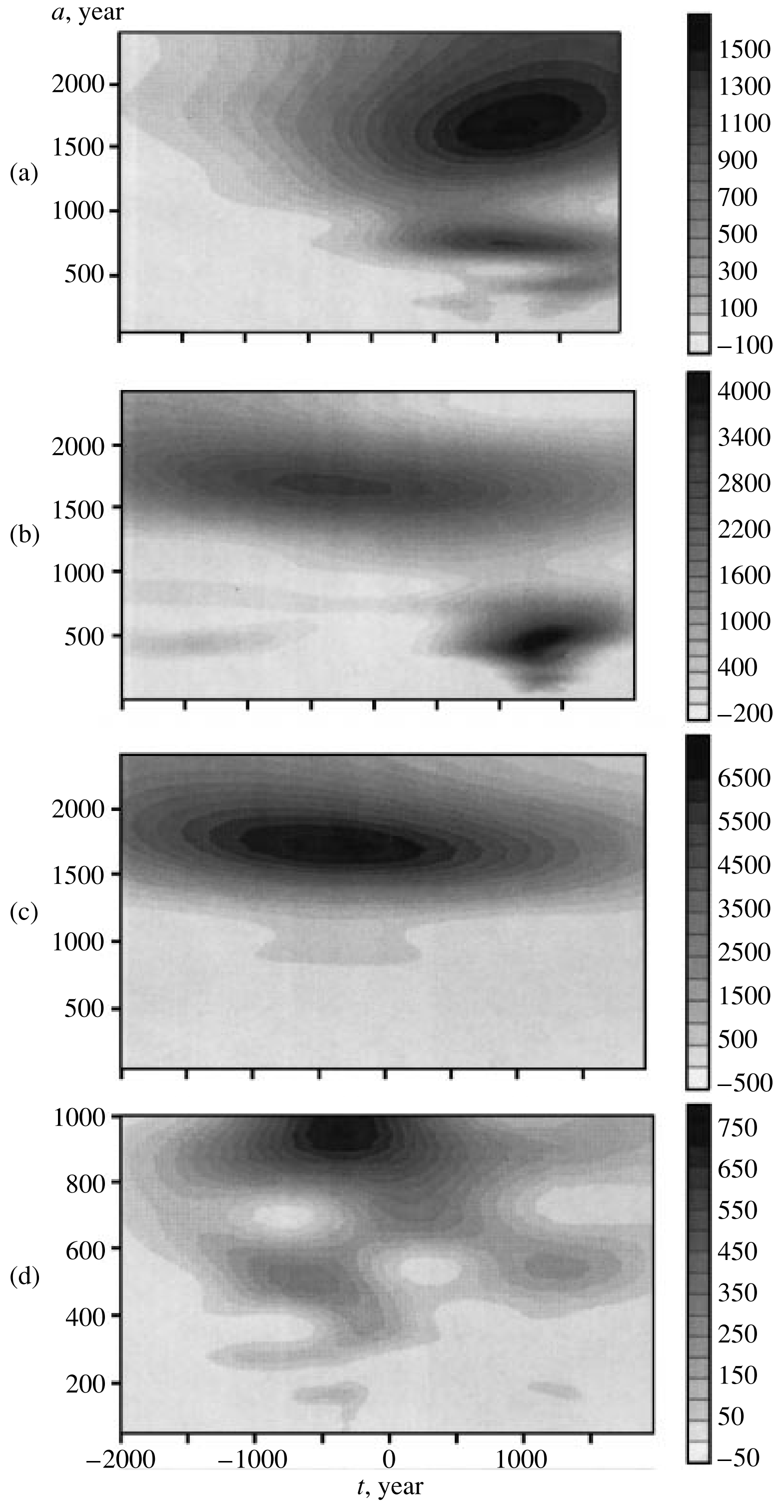

Fig. 3. Behavior of the spectrum coefficients of adaptive wavelets $w(a, t)$ for (a) Bulgaria, (b) Georgia, and (c), (d) Central Asia.

data for Central Asia. Note that the primary dissimilarity of the adaptive spectra from the wavelet spectra is that the positions of maximums in a range of about 1750 yr are almost coincident.

The obtained spectra are much more smooth than those derived on the basis of the Fourier transformation, although it also gives the characteristic times of oscillations (1560 yr for Bulgaria and 1770 yr for Geor-

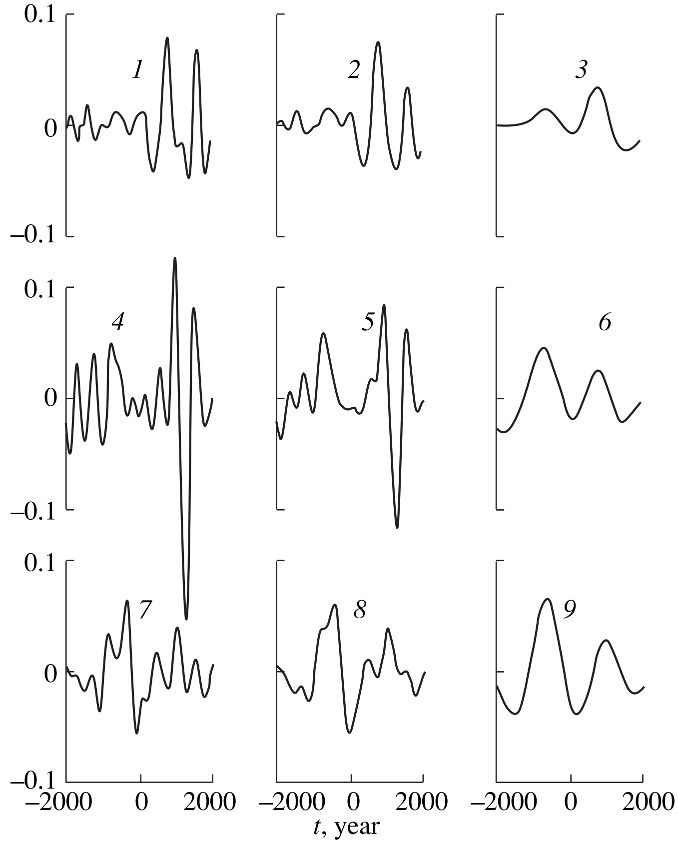

Fig. 4. Behavior of coefficient $w(a = \text{const}, t)$ in the "Mexican hat" wavelet expansion. (*1*)–(*3*) Bulgaria, for a = 500, 700, and 1750 years, respectively, (*4*)–(*6*) Georgia, for a = 500, 700, and 1750 years, respectively, (*7*)–(*9*) Central Asia, for a = 500, 700, and 1750 years, respectively.

gia and Central Asia [7]) close to our estimates obtained in the framework of the wavelet analysis.

The distribution of the square of the wavelet coefficient modulus is shown in Fig. 3, where the process with a time constant of 1750 yr, represented by the horizontal band in all of the three spectra, can be referred to periodic processes in the time segment of interest. (The distribution of the wavelet coefficients for Central Asia is detailed in Fig. 3d). The 750-year variation is also traced for the past two thousand years in the data for Bulgaria. The variation of about the same period is followed, albeit, less confidently in Georgia over all four millennia. The 900–1000-year variation exists in the entire time interval for Central Asia, but it manifests itself most clearly in the I millennium B.C.–the first half of the I millennium A.D. The amplitude of the nominal 500-year variation (the period of this variation is somewhat shorter in Bulgaria and somewhat longer in Central Asia) either enhances or weakens over four thousand years in all of the territories. The fact that the nominal 700- and 500-year variations are identified in Bulgaria only in the past two millennia is related to the paucity of data at the beginning of the observational interval. The interruptions in the manifestation of the 500-year variation in Georgia and Central Asia cannot be due to insufficient information on the field intensity variation, since the time intervals in which the variation is not identified in these regions are parts where the curves of the field intensity variation is obtained in greatest detail. The time interval in which the 500-year variation in Georgia attenuates is the second half of the I millennium B.C. to the beginning of A.D., and that in Central Asia covers the first three quarters of the I millennium A.D. In both cases, the duration of the intervals is almost the same, but the variation in Georgia attenuates earlier.

Figure 4 illustrates the behavior of coefficients $w(a = \text{const}, t)$ for the Mexican hat wavelet, with a corresponding to the nominal 500-, 700-, and 1750-year variations in Bulgaria, Georgia, and Central Asia. For convenient comparison between the amplitudes of different-duration oscillations, we use the normalization $\sigma = 1$, for which the wavelet coefficients are measured in units proportional to the units of signal measurement. The results obtained for Bulgaria in a time interval of B.C. will not be discussed here because of the insufficiently representative data involved. The most noticeable feature of the different-period variations in the various regions is a great similarity in the manifestation of the 700- and 500-year variations. The amplitude of the 1750-year variation in both Georgia and Central Asia in a period of B.C. is greater than that in a period of A.D., the amplitude in Georgia being smaller than in Central Asia. The amplitudes of the 700- and 500-year variations in Georgia and Bulgaria are much higher for the A.D. era than those for the B.C. era, and the data for Central Asia are more likely to indicate a larger span of oscillations in the B.C. period compared to the A.D. period. Also, the amplitudes of these variations in Bulgaria and Georgia (during the A.D. era) are substantially greater than those in Central Asia. We call attention to an abrupt spike of the wavelet coefficient for Georgia near the tenth century A.D. The spike corresponds to an abrupt drop of curve 1b (see the corresponding spot) in the tenth century A.D. Some of the noted features in the course of the geomagnetic field variations in various N–S sectors were already mentioned [7, 8]. Specifically, in studying the field variations in Central Asia over the two past millennia, it was shown that the variation with a period of about 500 years is confidently identified [9], and the variation with a period of about 500 years has the greatest amplitude (however, it is not identified significantly). When the data on the field variations in the I millennium B.C. [8] are taken into consideration, the amplitude of the 700-year variation sharply increases permitting its reliable identification. The disturbances in the field, i.e., variations in the span of its intensity oscillations with time (in a part containing the variations with a period less than 600 years) were studied from data on the change in the intensity in Georgia and Bulgaria over the past seven millennia [11]. It was established that the disturbance pulsates with time. The maximum amplitude of the oscillations is reached in the mid-II millennium B.C., and the minimum amplitude is observed near the turn of the A.D. era.

We see that the results of our wavelet analysis are reasonably consistent with the Fourier analysis data, but the wavelet analysis makes it possible to follow the continuous variation in the field characteristics with time.

Let us define the complex-valued correlation coefficient of processes $f_1(t)$ and $f_2(t)$, having the corresponding wavelet representations $w_1(a, t)$ and $w_2(a, t)$, in the form

$$C(a) = \frac{\int w_1(a,t) w_2^*(a,t) dt}{\left(\int |w_1(a,t)|^2 dt \int |w_2(a,t)|^2 dt\right)^{1/2}}.$$

The degree of correlation between the two processes on a given timescale a determines the modulus of $C(a)$ that may take values from zero to unity. The phase of $C(a)$ determines the shift (in rad) for the corresponding spectrum components of two processes. Figure 5 presents the moduli of correlation coefficient $|C(a)|$ calculated for three pairs of data: (a) Bulgaria and Georgia, (b) Central Asia and Georgia, and (c) Central Asia and Bulgaria. One can see that the correlation drops as the oscillation period decreases.

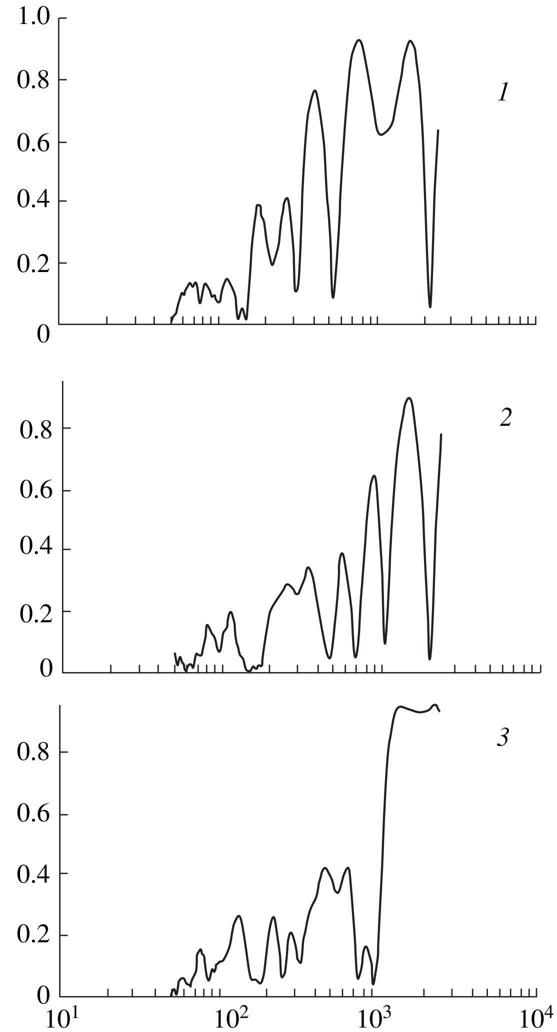

Fig. 5. Modulus of the correlation coefficient of the wavelet expansions. (*1*) Bulgaria and Georgia, (*2*) Central Asia and Bulgaria, (*3*) Central Asia and Georgia.

The phase shift for the 1750-year cycle that is identified in all of the data series can be found by using the phase of the correlation coefficient. The obtained values are 16° for Bulgaria and Georgia, 21° for Georgia and Central Asia, and 37° for Bulgaria and Central Asia (the latter value is the sum of the previous ones, which agrees with the hypothesis of the unified nature of the 1750-year cycle). Using the given geographic longitudes of the three examined regions (whose latitudes are close to each other), we obtain the angular distance between them and found velocity $v \approx 0.2°$/yr of the corresponding wave propagating from the east to the west.

CONCLUSION

The above analysis of stability of the time spectra showed that the 1750-year variation can confidently be identified in the data available.

The oscillations with periods of 700 and 500 years are traced in all of the regions examined, but their characteristics change with time. The moment of "attenuation" of the 500-year variation in Georgia comes earlier than in Central Asia, which again corroborates the presence of the shift in the pattern of geomagnetic field intensity on the Earth's surface, depending on the longitude of the region.

Application of the wavelet analysis allows us to follow the change in the character of the variations and the degree of manifestation of individual oscillations in different time intervals. This opens the possibility to study the morphology of geomagnetic field variations in greater detail than considered before.

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REFERENCES

1. Petrova, G.N., Nechaeva, T.B., and Pospelova, G.A., *Kharakternye izmeneniya geomagnitnogo polya v proshlom* (Characteristic Variations in the Past Geomagnetic Field), Moscow: Nauka, 1992.
2. Burlatskaya, S.P., *Arkheomagnetizm. Izuchenie drevnego geomagnitnogo polya* (Archeomagnetism: Study of the Ancient Geomagnetic Field), Moscow: Mir, 1987.
3. Kanasevich, E.R., *Analiz vremennykh posledovatel'nostei v geofizike* (Analysis of Temporal Sequences in Geophysics), Moscow: Nedra, 1985.
4. Astaf'eva, N., Wavelet Analysis of Astronomical Data, *Usp. Fiz. Nauk*, 1996, no. 11.
5. Frick, P., Baliunas, S.L., Galyagin, D., and Soon, W.H., Wavelet Analysis of Stellar Chromospheric Activity Variations, *Astrophys. J.*, 1997, vol. 483, p. 426.
6. Galyagin, D.K. and Frick, P.G., Adaptive Wavelets (Algorithms for the Spectral Analysis of Signals with Gaps in Data), *Mat. model. sist. protsessov*, 1996, no. 4, p. 10.
7. Nachasova, I.E. and Burakov, K.S., Geomagnetic Field Intensity Variations in the Past 4000 Years from World Data, *Dokl. Ross. Akad. Nauk*, 1997, vol. 353, no. 2, p. 255.
8. Nachasova, I.E. and Burakov, K.S., Geomagnetic Field Intensity in Central Asia in the I–II Millenniums B.C., *Fiz. Zemli*, 1997, no. 7, pp. 33–38.
9. Nachasova, I.E. and Burakov, K.S., Variations in the Central Asian Geomagnetic Field for the Past 2000 Years: Analysis of World Data, *Geomagn. Aeron*., 1995, no. 6, p. 150.
10. Daubechies, I., *Ten Lectures on Wavelets*, Phelodelphia: FIAM, 1992.
11. Nachasova, I.E. and Burakov, K.S., Variation in the Disturbance of Geomagnetic Field Intensity in the Past 7000 Years, *Geomagn. Aeron*., 1997, no. 6, p. 150.